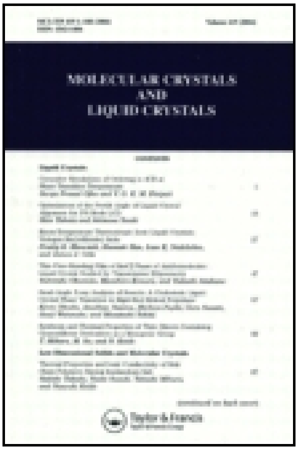

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J. T. Jenkins [a]
[a] Department of Theoretical and Applied Mechanics, Cornell University, Ithaca, New York
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Propagating Plane Disinclination Surfaces in Nematic Liquid Crystals

J. T. JENKINS

Department of Theoretical and Applied Mechanics
Cornell University
Ithaca, New York

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We indicate how the use of the integral from of the balance of moment of momentum leads to a simplification of a theory of propagating plane disinclination surfaces in nematic liquid crystals proposed by Currie.[1]

INTRODUCTION

Recently, Currie[1] has discussed the possible existence of propagating disinclination surfaces in nematic liquid crystals. Across these surfaces the orientation in the liquid crystal and the acceleration of the fluid suffer a discontinutity while the velocity of the fluid is continuous. His analysis of the wave was complicated by the presence of a vector β which arises from the assumption that the director, describing the orientation of the material, has fixed magnitude. Here we show that the analysis is simplified when the jump condition resulting from the integral form of the balance of moment of momentum is taken into account.

GOVERNING EQUATIONS

As does Currie, we adopt the integral forms of the balance laws for energy, linear momentum and director inertia proposed by Leslie[2] for incompressible nematic liquid crystals; however, we emphasize the integral form of the balance law for moment of momentum.[3] Leslie writes down only a local form of this balance law. The director $d\,(x,t)$ and the velocity $v(x,t)$ are constrained by

$$d_i d_i = 1, \quad d_i d_{i,j} = 0, \quad d_i \dot{d}_i = 0, \quad v_{i,j} = 0, \tag{2.1}$$

where the dot denotes the material time derivative.

The balance laws for a material volume V bounded by a surface A are

$$\frac{d}{dt} \int_V \left(\frac{1}{2}\rho\, v_i v_i + \frac{1}{2}\rho_1 \dot{d}_i \dot{d}_i + W + TS \right) dV = \\ = \int_V (r + F_i v_i + G\, \dot{d}_i)\, dV + \int_A (v_i \sigma_{ij} + \dot{d}_i \pi_{ij} - q_i)\, \nu_j\, dA, \tag{2.2}$$

$$\frac{d}{dt} \int_A \rho v_i dV = \int_A F_i dV + \int_A \sigma_{ij} \nu_j dA, \tag{2.3}$$

$$\frac{d}{dt} \int_V \rho_1 \dot{d}_i dV = \int_V (g_i + G_i)\, dV + \int_A \pi_{ij} \nu_j dA, \tag{2.4}$$

and

$$\frac{d}{dt} \int_V (\rho x_{[i} v_{j]} + \rho_1 d_{[i} \dot{d}_{j]}) dV = \int_V (x_{[i} F_{j]} + d_{[i} G_{j]})\, dV + \\ + \int (x_{[i}\sigma_{j]k} + d_{[i}\pi_{j]k}) \nu_k dA \tag{2.5}$$

Here ρ is the constant density, ρ_1 a positive inertial constant, T is the temperature, r is the energy supply, F the external body force, G the external director force, and ν is the unit outward normal to A. Brackets about two indicies indicate that these indices should be antisymmeterized. The free energy W, the entropy S, the stress tensor σ, the director stress π, the heat flux vector q and the intrinsic body force g are given by constitutive equations[2]. Of these, we require only

$$W = W\,(T, d_i, d_{i,j}) \tag{2.6}$$

and

$$\pi_{ij} = d_i \beta_j + \frac{\partial W}{\partial d_{i,j}} \quad , \tag{2.7}$$

where β is an undetermined vector included in the theory because of the constraint on the magnitude of the director. We assume that the form of the free energy is such that

$$d_{i,j} = 0 \tag{2.8}$$

implies that

$$\frac{\partial W}{\partial d_{i,j}} = 0. \tag{2.9}$$

In regions where the fields are sufficiently smooth, local expressions of the balance of energy, linear momentum, director inertia and moment of momentum may be written down. The local form of the balance of moment of momentum is usually required to be satisfied as an identity, leading to restrictions on the constitutive equations.[2]

JUMP CONDITIONS

Following Currie, we consider a singular surface across which ν and T are continious but d and derivatives of $d.\nu$ and T are discontinuous. The surface is assumed to be plane and propagating with velocity U into a region in which ν is zero and d and T are constant. The unit normal to the surface, pointing in the direction of propagation, is n. Brackets enclosing a quantity indicate the difference, called the jump, in the values of that quantity ahead of and behind the wave.

As a consequence of the balance laws, we have the following jump conditions across the singular surface (Truesdell and Toupin[4], secs. 178-181):

$$[W + TS + \frac{1}{2}\,\rho_1\,\dot{d}_i\dot{d}_i]\,U + [v_i\sigma_{ij} + \dot{d}_i\pi_{ij} - q_j]\,n_j = 0, \tag{3.1}$$

$$[\sigma_{ij}]\,n_j = 0, \tag{3.2}$$

$$[\rho_1\,\dot{d}_i]\,U + [\pi_{ij}]\,n_j = 0, \tag{3.3}$$

and

$$[\rho_1\,d_{[i}\dot{d}_{j]}]\,U + [d_{[i}\pi_{j]k}]n_k = 0. \tag{3.4}$$

We note that, because at the outset Currie did not consider the integral form of the balance law of moment of momentum (2.5), he does not obtain the jump condition (3.4) resulting from this law.

Currie next writes down compatibility conditions which must be satisfied by the jumps in the derivatives of ν, d and T in order for the surface of discontinuity to persist. Then, respecting the constraints (2.1), he introduces into the jump conditions constitutive equations for W,S,q,g,π, and σ, compatible with the local moment of momentum identity, and determines under what conditions and with what speed a singular surface will propagate without decay through the material.

Currie obtains propagation conditions which depend upon the values of $\beta \cdot n$ ahead of and behind the wave. To allow for all possibilities, he treats four cases

(i) $[\beta \cdot n] = 0, \beta_0 = 0$
(ii) $[\beta \cdot n] = \text{arbitrary}, \beta_0 = 0$
(iii) $[\beta \cdot n] = 0, \beta_0 \neq 0$
(iv) $[\beta \cdot n] = \text{arbitrary}, \beta_0 \neq 0$,

where β_0 is the value of $\beta \cdot n$ ahead of the wave. We show next that the last two cases need not be considered.

In order to obtain this simplification, we exploit the jump condition (3.4) resulting from the integral form of the balance law for moment of momentum. We use the superscripts "plus" and "minus" to denote the values of a quantity ahead of and behind the surface, respectively, and writes

$$D_i \equiv [d_i] = d_i^+ - d_i^-. \tag{3.6}$$

Then, as a consequence of (3.3), the left hand side of (3.4) may be written as

$$\begin{aligned} [d_{[i}(\rho_1 \dot{d}_{j]} U + \pi_{j]k} n_k)] &= D_{[i}(\rho_1 d_{j]}^- U + \pi_{j]k}^- n_k) \\ &= D_{[i}(\rho_1 d_{j]}^+ U + \pi_{j]k}^+ n_k) . \end{aligned} \tag{3.7}$$

But d^+ is a constant, which implies that

$$\dot{d}_i^+ = 0, \quad \pi_{ij}^+ = d_i^+ \bar{\beta}_j^+ ; \tag{3.8}$$

so

$$D_{[i} d_{j]}^+ (\beta_k^+ n_k) = d_{[i}^- d_{j]}^+ (\beta_k^+ n_k) = 0 \tag{3.9}$$

Now it has already been assumed that d suffers a discontinuity across the surface, hence

$$d_{[i}^- d_{j]}^+ \neq 0. \tag{3.10}$$

Thus

$$\beta_0 \equiv \beta_k^+ n_k = 0, \tag{3.11}$$

eliminating cases (iii) and (iv).

In addition, we need not consider case (i) separately, for the solutions of the propagation condition of case (ii) include those of case (i). That is, Currie's undamped waves (4.5) with $d^- \cdot n = 0$. and (4.6) satisfy his propagation condition (4.4) for case (ii) and give $\beta^- \cdot n = 0$.

Finally, we recall that in order to obtain these simplifications of Currie's results we utilized jump conditions resulting from the four balance laws (2.2)-(2.5). In fact, using Currie's methods one may obtain the simplified from of his results by beginning with the three balance laws for the conserved quantities-linear momentum, moment of momentum and energy. With this approach the arbitrary fields associated with the constraint on the director do not enter into the analysis and, for example, one obtains directly a propagation condition which is Currie's equation (4.4). In other words, that part of the jump condition resulting from the director balance which is not satisfied as a consequence of the balance of moment of momentum has no physical significance and serves only to determine $\beta^{-}\cdot n$.

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